# Growth of a tensionless interface in anisotropic random media

Kwangho Park
*Department of Mathematics and Statistics, Arizona State University, Tempe, Arizona 85287, USA*

Aeran Ji, Jae Hwan Lee, and In-mook Kim*
*Department of Physics, Korea University, Seoul 136-701, Korea*
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We introduce a simple growth model where a tensionless interface grows in random media. In this model, the degree of anisotropy of the random media is controlled by a variable $g$. When $g=0$, there is no anisotropic property of the random media. But, the anisotropic property increases as $g$ does from 0. From the numerical simulations, we find that this model belongs to the quenched Herring-Mullins universality class when $g=0$. Interestingly, however, we find that this model belongs to the quenched Kardar-Parisi-Zhang universality class when $g$ is nonzero.

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The study of a growing interface in random media has been a popular research topic for the last 20 years because it relates to various physical systems such as interface growth in porous media [1,2], charge density waves under external fields [3–5], fluid imbibition in paper [6], driven flux motion in type-II superconductors [7,8], etc.

The growth velocity of an interface driven in random media is determined by an external driving force $F$. The growth velocity is zero when the driving force $F$ is smaller than the pinning strength induced by the quenched disorder of the random media. There exists a threshold of the driving force $F_c$ above which the interface moves with a constant velocity. Accordingly, the velocity is zero for $F<F_c$, and it is nonzero for $F>F_c$. This phenomenon is called the pinning-depinning (PD) transition. Near the depinning threshold, the depinned interface shows nontrivial scaling behavior in the global interface width [9],

$$W(L,t)=\left\langle \frac{1}{L^{d'}} \sum_x [h(x,t)-\overline{h}(t)]^2 \right\rangle^{1/2}, \tag{1}$$

where $h(x,t)$ is the height of the interface at position $x$ and time $t$. $L$, $d'$, and $\overline{h}$ denote the system size, the substrate dimension, and the mean height, respectively. The interface width scales as

$$W(L,t)\sim\begin{cases} t^{\beta} & \text{if} \quad t\ll L^z \\ L^{\zeta} & \text{if} \quad t\gg L^z. \end{cases} \tag{2}$$

The exponents $\zeta$, $\beta$, and $z$ are called the roughness, the growth, and the dynamic exponents, respectively. These exponents are related by $z\beta=\zeta$. It is well known [9] that the dynamical behaviors of the fluctuating interfaces can be classified into a finite number of universality classes by the exponents $\zeta$, $\beta$, and $z$.

The driven interface in random media shows different dynamical behavior according to the property of the random media [9,10]. In isotropic random media, the growth velocity of a driven interface does not depend on the slope $s$ of a tilted substrate near the depinning threshold or becomes independent of $s$ at the depinning threshold although there is the dependence of the growth velocity on $s$ far from the depinning threshold. In anisotropic random media, however, the growing velocity of a driven interface depends on $s$ even at the depinning threshold. There have been many studies about the dynamical behavior of a driven interface in isotropic/anisotropic random media, where the growth of the interface is affected by interface tension [9]. Recently two independent studies about the growth of a tensionless interface driven in isotropic random media are also reported [11,12], but there has been no study in anisotropic random media. It would be thus interesting to study the growth of a tensionless interface driven in anisotropic random media and to classify the universality class of the dynamics.

Depinning dynamics of a driven interface in isotropic random media can be explained by a Langevin-type continuum equation, the quenched Edwards-Wilkinson (QEW) equation [9,13],

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h(x,t)+\eta(x,h)+F, \tag{3}$$

where $\nu\nabla^2 h(x,t)$ describes the smoothening effect of interface tension. $\eta(x,h)$ is a quenched noise with $\langle \eta(x,h)\rangle=0$ and $\langle \eta(x,h)\eta(x',h')\rangle=2D\delta^{d'}(x-x')\delta(h-h')$. The quenched noise term describes a random force by isotropic quenched disorder in random media. The QEW equation shows a PD transition. In the QEW equation, the growth velocity of the interface near the depinning threshold does not depend on the slope of a tilted substrate.

A few years ago, Park *et al.* [14] studied the interface growth in random media with a simple growth model mimicking the interface growth at the depinning threshold. The growth rule of the model is defined as follows.

(i) A random number between 0 and 1 is assigned on each lattice site in one-dimensional system, where the random numbers represent impurities of random media.

*Corresponding author. Electronic address: imkim@korea.ac.kr

 

(ii) At each time $t$, the local force $f_i(t)$ is calculated for each site $i$,

$$f_i(t)=h_{i+1}(t)-2h_i(t)+h_{i-1}(t)+m[1+gs_l(i)]\eta_{i,h_i}, \tag{4}$$

where $m$ and $g$ are integers. $h_i(t)$ denotes the height at time $t$ and a site $i$. $\eta_{i,h_i}$ denotes the random number at a site $i$ and the height $h_i$. The local slope is $s_l(i)=1$ only when $h_{i+1}-h_i>0$ or $h_{i-1}-h_i>0$, otherwise $s_l(i)=0$. When $g=0$, there is no anisotropic property of the random media. But as $g$ increases from 0, the degree of the anisotropic property increases.

(iii) The column having the maximum $f_{\max}\equiv\max[f_i]$ among all $f_i$ increases as follows:

$$h_i(t+1)=h_i(t)+1 \quad \text{if} \quad f_i=f_{\max},$$

$$h_i(t+1)=h_i(t) \quad \text{otherwise.} \tag{5}$$

In this model, the interface tends to grow faster at the site where the local slope $s_l$ is 1 when $g>0$. The dynamical behavior of the interface in this model is affected by the slope $s$ of the tilted substrate if $g>0$. The movement of the driven interface in this model can be described by the EW equation with general type of a quenched noise term instead of an isotropic quenched noise term,

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h(x,t)+\tilde{\eta}(x,h)+F, \tag{6}$$

where the generalized quenched noise satisfies the conditions, $\langle\tilde{\eta}(x,h)\rangle=0$ and $\langle\tilde{\eta}(x,h)\tilde{\eta}(x',h')\rangle=2D[1+f(s_l)]\delta^{d'}(x-x')\delta(h-h')$. Here, $f(s_l)$ is a function depending on the local slope $s_l$. It was shown by simple calculation that the growth velocity of Eq. (6) at the depinning threshold depends on the slope $s$ of the tilted substrate. Park *et al.* showed from computer simulations that their model exhibits the same dynamical scaling behavior at the depinning threshold for $g>0$ as the quenched Kardar-Parisi-Zhang (QKPZ) equation [9,15],

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h(x,t)+\frac{\lambda}{2}[\nabla h(x,t)]^2+\eta(x,h)+F. \tag{7}$$

The QKPZ equation shows a PD transition. The interface velocity at the depinning threshold in the QKPZ equation depends on the slope $s$ of the tilted substrate because of the second term on the right-hand side in Eq. (7).

The depinning dynamics of a driven tensionless interface in isotropic random media can be well explained by a Langevin-type continuum equation, the quenched Herring-Mullins (QHM) equation [16],

$$\frac{\partial h(x,t)}{\partial t}=-\kappa\nabla^4 h(x,t)+\eta(x,h)+F. \tag{8}$$

From the numerical studies [11,12], it was known that the QHM equation shows different dynamical behavior at the depinning threshold from that of the QEW and the QKPZ equations.

In this paper, we introduce a simple self-organized automaton model (SOAM), which mimics the growth of a driven tensionless interface in anisotropic random media at the depinning threshold. In this model, one can control the degree of the anisotropy of the medium.

The growth rule of our model is as follows. First, we assign a random number between 0 and 1 on each lattice site in one-dimensional system, where the random numbers represent the impurities of random media. Second, at each update, we calculate the local force for each site $i$,

$$f_i=-\kappa(h_{i+2}-4h_{i+1}+6h_i-4h_{i-1}+h_{i-2})$$
$$+[1+gs_l(i)]\eta_{i,h_i}, \tag{9}$$

where $\kappa$ and $g$ are integers. $\eta_{i,h_i}$ denotes the random number at a site $i$ and the height $h_i$. $L$ updates correspond to a time increment of 1. The local slope is $s_l(i)=1$ only when $h_{i+1}-h_i>0$ or $h_{i-1}-h_i>0$, otherwise $s_l(i)=0$. Third, the growth process of the interface is the same as Eq. (5).

When $g=0$, the dynamical behavior of the model is well described by the QHM equation with an isotropic quenched noise term at the depinning threshold. The obtained roughness exponent is $\zeta=2.28(5)$ for $\kappa=0.1$ and $g=0$ (see the inset of Fig. 2). Through some simulations for different values of $\kappa$, we found that the value of the roughness exponent does not depend on the value of $\kappa$. Therefore, we used a fixed value $\kappa=0.1$ in our simulations. Our SOAM model can be described generally by the following continuum equation at the depinning threshold $F=F_c$:

$$\frac{\partial h(x,t)}{\partial t}=-\kappa\nabla^4 h(x,t)+\tilde{\eta}(x,h)+F. \tag{10}$$

We carried out computer simulations of our SOAM model for $g=20$, 40, 60, 80, and 100. Numerical data were averaged typically over more than 200 configurations. In order to obtain the growth exponent $\beta$, we measured the time-dependent behavior of the interface width $W(L,t)$ starting from an initially flat interface. As shown in Fig. 1, the value of the growth exponent $\beta$ decreases as $g$ increases. We plot the growth exponent $\beta$ versus $1/g$ in the inset of Fig. 1. We found that the value of $\beta$ approaches 0.66(2) as $1/g$ does 0. The value $\beta=0.66(2)$ is in good agreement with that in the QKPZ universality class [9,17]

When $g=0$, we could not measure the reliable value of $\beta$ since it varies as time goes on. We thus tried to estimate $\beta$ from the saturated interface width [17]. If the width of the interface is saturated at time $t_s$, the new interface height $\tilde{h}_i(t)$ is defined as $h_i(t+\tau)-h_i(\tau)$, where $\tau>t_s$. The interface width obtained from $\tilde{h}_i(t)$ shows the scaling behavior $W(L,t)\sim t^{\beta_s}$ before saturation. We obtained $\beta_s=0.75(1)$, which is in good agreement with the result obtained from the previous study about the QHM equation with an isotropic quenched noise term [12].

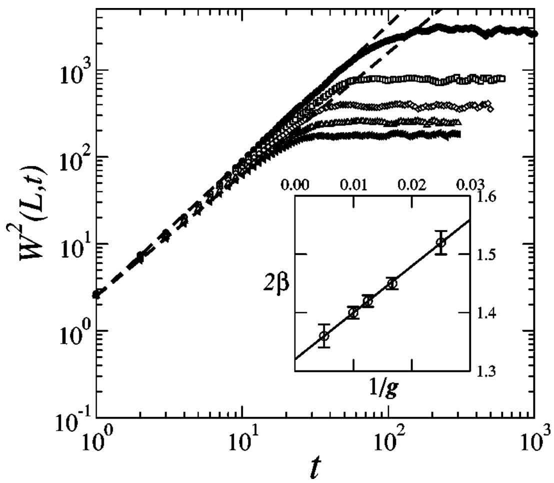


FIG. 1. The plots of the interface width as a function of time for $g=20$, 40, 60, 80, and 100 with the system size $L=16\,384$. The slopes of two dashed lines are for $2\beta=1.60$ and 1.40 for $g=20$ and 100, respectively. Inset: The plot of $\beta$ as a function of $1/g$ for $g=40$, 60, 80, 100, and 200. $\beta$ approaches 0.66 as $1/g$ does 0.

In order to obtain the roughness exponent, we plot the saturated value of $W^2(L)$ versus the system size $L$ in double logarithmic scales in Fig. 2. We obtained $\zeta=2.28(5)$ in the QHM universality class when $g=0$ (see the inset of Fig. 2) and $\zeta=0.64(1)$ in the QKPZ universality class when $g\geqslant 80$ [9,17]. Although we could not estimate the correct value of the roughness exponent $\zeta$ for $g=10$, 20, 40, and 60 because of crossover behavior, we found that the local slope of the interface width decreases from large value and approaches 0.64 as the system size increases.

We also measure the height-height correlation function $C(x)$ defined as

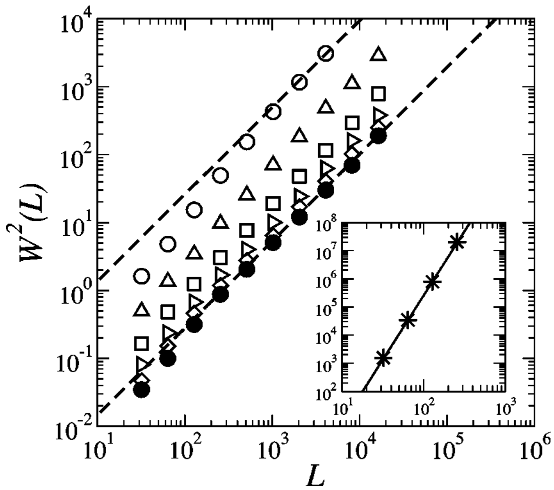


FIG. 2. The plots of $W^2(L)$ at the saturated regime as a function of $L$ in double logarithmic scales are shown for $g=10$, 20, 40, 60, 80, and 100 from top to bottom. The system sizes $L=32$, 64, 128, 256, 512, 1024, 2048, 4096, 8192, and 16 384. The dashed guide lines are for $2\zeta=1.28$. Inset: The plot of $W^2(L)$ at the saturated regime vs $L$ for the system sizes $L=32$, 64, 128, and 256 when $g=0$. The slope of the straight line is $2\zeta=4.56$.

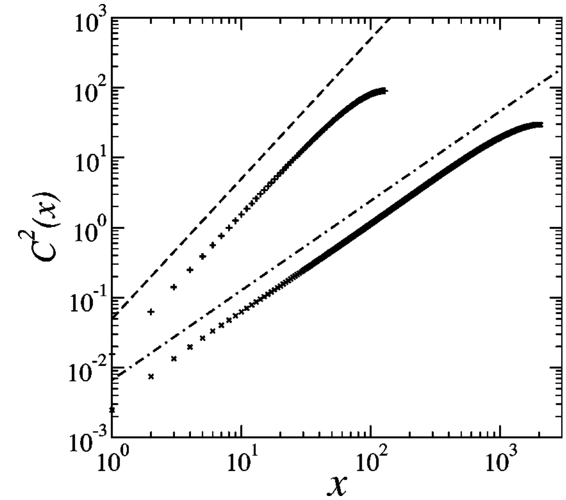


FIG. 3. The plot of the height-height correlation function $C^2(x)$ vs $x$ is shown for $g=0$ (top) and $g=10$ (bottom) with the system sizes $L=256$ and 4096, respectively. The top straight line represents $2\zeta'=2$ and the bottom does $2\zeta'=1.28$.

$$C(x)=\left\langle \frac{1}{L^{d'}}\sum_{x}\,[h(x+x_1,\tau)-h(x_1,\tau)]^2\right\rangle^{1/2}, \qquad (11)$$

where $\tau$ is larger than the saturation time $t_s$ and $C(x)$ scales as $x^{\zeta'}$. The value of the roughness exponent measured from $C(x)$ is $\zeta'=1.00(1)$ when $g=0$ and $\zeta'=0.64(1)$ when $g=10$ (see Fig. 3). The values of $\zeta'$ obtained when $g=0$ and 10 are in good agreement with those in the QHM and the QKPZ universality class, respectively. When $g=0$, the value of $\zeta'$ is smaller than $\zeta=2.28$ obtained from the interface width. It is well known that the anomalous scaling of the local width is due to the superroughening [18,19], in such a way that the roughness exponent $\zeta'$ obtained from the height-height correlation function is smaller than $\zeta$ obtained from the saturated value of $W^2(L,t)$. The superrough scaling occurs only when the roughness exponent $\zeta$ is larger than 1. Therefore, the two roughness exponents $\zeta$ and $\zeta'$ have different values at $g=0$. When $g>0$ and $L$ is large, the superrough scaling behavior does not appear because $\zeta$ is smaller than 1, i.e., $\zeta'=\zeta$. When $g=10$ and $L=4096$, the roughness exponent $\zeta'$ shows a crossover behavior from $\zeta'>0.64$ for small values of $x$ to $\zeta'=0.64(1)$ for the large values of $x$. This result supports the fact that our model has $\zeta=\zeta'=0.64(1)$ when $g>0$ and so belongs to the QKPZ universality class.

The relaxation function method is known to be useful for measuring the dynamic exponent $z$ independently [12,20]. We prepare a sinusoidal initial interface described as

$$h(x,0)=A\sin(2\pi x/l), \qquad (12)$$

where $A$ and $l$ are the amplitude and the wavelength, respectively. The interface evolves following the growth rule of our SOAM model on this initial interface. When an interface grows, we measure the normalized relaxation function $R(t,l)$. $R(t,l)$ is defined as

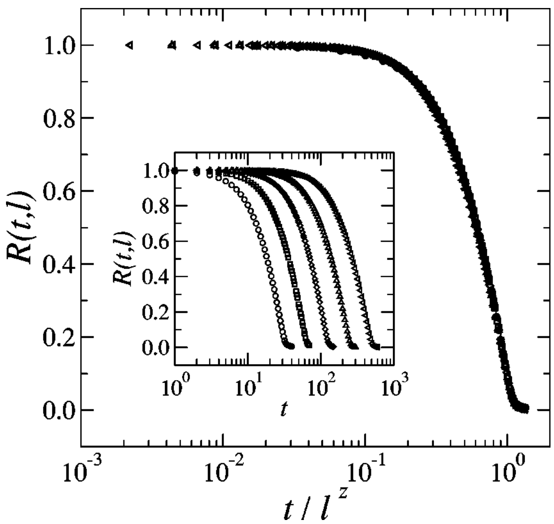


FIG. 4. The data collapse of the relaxation functions shown in the inset with $z=0.98$ when $L=8192$. Inset : The relaxation function $R(t,l)$ vs $t$ for $l=32$, 64, 128, 256, and 512 from the left.

$$R(t,l)=C_A(t,l)/C_A(0,l), \tag{13}$$

where $C_A(t,l)$ is an autocorrelation function of the height, defined by $C_A(t,l)=\langle h(x,0)h(x,t)\rangle$. $R(t,l)$ follows the scaling form

$$R(t,l)\sim f(t/l^z), \tag{14}$$

where $f$ is a universal scaling function. $R(t,l)$ is shown in the inset of Fig. 4, where $R(t,l)$ was measured when $g=10$ and $L=8192$. All curves collapse well into a universal curve with $z=0.98(2)$ as shown in Fig. 4. We also obtained the same result for $g\geqslant 20$. According to the scaling relation $z\beta=\zeta$, one can find that the growth exponent $\beta$ for $g=10$ is 0.65(3) from $z=0.98(2)$ and $\zeta=0.64(1)$. This result is also in good agreement with that of the previous studies about the model in the QKPZ universality [9,17]. In our SOAM model, the interface tends to grow faster at the site $i$ where the local slope $s_l(i)$ of the interface is nonzero. Therefore, the interface growth in this model is affected by the slope $s$ of the tilted substrate. Such slope dependence is known to induce the KPZ nonlinear term during the interface growth [14], i.e., $\partial h(x,t)/\partial t\sim\lambda/2(\boldsymbol{\nabla} h)^2$. The interface in our model tends to grow well at the site $i$ where the random number is small. This effect plays a role of the isotropic quenched noise term during the interface growth [14], i.e., $\partial h(x,t)/\partial t\sim\eta(x,h)$. The interface in our model also tends to grow well at the site $i$ where the condition $h_{i+1}-2h_i+h_{i-1}=\nabla^2 h_i>0$ is satisfied. This type of updates makes the surface tension effect occur during the interface growth, i.e., $\partial h(x,t)/\partial t\sim\nabla^2 h(x,t)$ [21]. Therefore, the dynamical behavior of our model for $g>0$ can be described effectively by the continuum equation

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h-\kappa\nabla^4 h+\frac{\lambda}{2}(\boldsymbol{\nabla} h)^2+\eta(x,h)+F, \tag{15}$$

where the terms $\nu\nabla^2 h$, $(\lambda/2)(\nabla h)^2$, and $\eta(x,h)$ are effectively induced from the anisotropic quenched noise $\tilde{\eta}(x,h)$. In the above equation, the effect of the term $-\kappa\nabla^4 h(x,t)$ in dynamical scaling behavior does not appear in the large system because of the surface tension term $\nu\nabla^2 h(x,t)$ [21]. The dynamical behavior in the growth of a tensionless interface in anisotropic random media is determined by three terms $\nu\nabla^2 h$, $(\lambda/2)(\boldsymbol{\nabla} h)^2$, and $\eta(x,h)$. Therefore, the dynamics of the tensionless interface driven in anisotropic random media belongs to the QKPZ universality class.

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